NAG Toolbox for MATLAB

s14ag

1 Purpose

s14ag returns the value of the logarithm of the Gamma function $\ln \Gamma(z)$ for complex z, via the function name.

2 Syntax

```
[result, ifail] = s14ag(z)
```

3 Description

s14ag evaluates an approximation to the logarithm of the Gamma function $\ln \Gamma(z)$ defined for $\mathrm{Re}(z) > 0$ by

$$\ln \Gamma(z) = \ln \int_0^\infty e^{-t} t^{z-1} dt$$

where $z = x + iy$ is complex. It is extended to the rest of the complex plane by analytic continuation unless $y = 0$, in which case z is real and each of the points $z = 0, -1, -2, \ldots$ is a singularity and a branch point.

s14ag is based on the method proposed by Kölbig 1972 in which the value of $\ln \Gamma(z)$ is computed in the different regions of the z plane by means of the formulae

$$\begin{aligned}
\ln \Gamma(z) &= \left(z - \tfrac{1}{2}\right) \ln z - z + \tfrac{1}{2} \ln 2\pi + z \sum_{k=1}^{K} \frac{B_{2k}}{2k(2k-1)} z^{-2k} + R_K(z) && \text{if } x \geq x_0 \geq 0, \\
&= \ln \Gamma(z+n) - \ln \prod_{\nu=0}^{n-1} (z+\nu) && \text{if } x_0 > x \geq 0, \\
&= \ln \pi - \ln \Gamma(1-z) - \ln(\sin \pi z) && \text{if } x < 0,
\end{aligned}$$

where $n = [x_0] - [x]$, $\{B_{2k}\}$ are Bernoulli numbers (see Abramowitz and Stegun 1972) and $[x]$ is the largest integer $\leq x$. Note that care is taken to ensure that the imaginary part is computed correctly, and not merely modulo 2π.

The function uses the values $K = 10$ and $x_0 = 7$. The remainder term $R_K(z)$ is discussed in Section 7.

To obtain the value of $\ln \Gamma(z)$ when z is real and positive, s14ab can be used.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Kölbig K S 1972 Programs for computing the logarithm of the gamma function, and the digamma function, for complex arguments *Comp. Phys. Comm.* **4** 221–226

5 Parameters

5.1 Compulsory Input Parameters

1: **z – complex scalar**

The argument z of the function.

Constraint: $\mathrm{Re}(\mathbf{z})$ must not be 'too close' (see Section 6) to a nonpositive integer when $\mathrm{Im}(\mathbf{z}) = 0.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – complex scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathrm{Re}(\mathbf{z})$ is 'too close' to a nonpositive integer when $\mathrm{Im}(\mathbf{z}) = 0.0$. That is, $\mathrm{ABS}(\mathrm{Re}(\mathbf{z}) - \mathrm{NINT}(\mathrm{Re}(\mathbf{z}))) < \textit{\textbf{machine precision}} \times \mathrm{NINT}(\mathrm{ABS}(\mathrm{Re}(\mathbf{z})))$.

7 Accuracy

The remainder term $R_K(z)$ satisfies the following error bound:

$$\begin{aligned} |R_K(z)| &\leq \frac{|B_{2K}|}{|(2K-1)|} z^{1-2K} \\ &\leq \frac{|B_{2K}|}{|(2K-1)|} x^{1-2K} \text{if } x \geq 0. \end{aligned}$$

Thus $|R_{10}(7)| < 2.5 \times 10^{-15}$ and hence in theory the function is capable of achieving an accuracy of approximately 15 significant digits.

8 Further Comments

None.

9 Example

```
z = complex(-1.5, +2.5);
[result, ifail] = s14ag(z)
```

```
result =
  -5.0140 - 4.0718i
ifail =
           0
```